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253. Proposed by HERBERT N. CARLETON, West Newbury, Mass.

Prove that $n^{2k+8} - n^{2k} \equiv 0 \pmod{20}$ for integral values of n and k.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

452. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Find in the form of a continued fraction the positive root of the equation $x^3 - 2x - 5 = 0$.

Solution by Horace Olson, Chicago, Illinois.

Since the left member of this equation is negative for $x = 2$, and positive for $x = 3$, the positive root of this equation can be written $2 + (1/x_2)$, where $x_2 > 1$. Substituting this expression for x, we obtain

$$x_2^3 - 10x_2^2 - 6x_2 - 1 = 0.$$

By trial, we find that this equation has a root between 10 and 11.

Putting $x_2 = 10 + (1/x_3)$, we obtain $61x_3^3 - 94x_3^2 - 20x_3 - 1 = 0$.

This process can be continued as far as may be desired. A convenient way of making the substitutions is to use Horner's method of solving equations and then to reverse the order of the terms of the equation resulting from each step. From the nature of the process it is evident that each of the successive equations has one, and only one, root greater than 1, since the original equation has one, and only one, root between 2 and 3. The root of the original equation will then be

$$2 + \frac{1}{x_1 +}\,\frac{1}{x_2 +}\,\frac{1}{x_3 +}\cdots.$$

This I find to be

$$2 + \frac{1}{10 +}\,\frac{1}{1 +}\,\frac{1}{1 +}\,\frac{1}{2 +}\,\frac{1}{1 +}\,\frac{1}{3 +}\,\frac{1}{1 +}\cdots.$$

Also solved by H. C. Feemster and J. W. Clawson.

453. Proposed by A. J. KEMPER, University of Illinois.

Is the series, whose terms are the reciprocals of all positive integers not containing the figure 0, convergent or divergent?

Solution by E. J. Moulton, Northwestern University.

I shall prove the following proposition:

The series S whose terms are the reciprocals of all positive integers not containing the digit 0 at least p times, where p is any positive integer, is convergent.

Let us group the terms of S according to the number of digits required in writing the integers; that is, into groups $g_1, g_2, \cdots$ where the denominators x_{r+1} of the terms in g_{r+1} satisfy the relation

$$10^r \leqq x_{r+1} < 10^{r+1}.$$

The number of terms in g_{r+1} can be shown to be

$$\text{(1)} \qquad N(r+1) = 9\cdot[\text{first } p \text{ terms of the expansion of } (9+1)^r]$$
$$= 9\cdot[9^r + {}_rC_1 9^{r-1} + {}_rC_2 9^{r-2} + \cdots + {}_rC_{p-1} 9^{r-p+1}].$$

When $r \geqq 2(p-1)$ we have ${}_rC_1 < {}_rC_2 < {}_rC_3 \cdots < {}_rC_{p-1}$; and hence,

$$N(r+1) \leqq 9\cdot {}_rC_{p-1}\cdot 9^r \cdot p = 9^{r+1}\cdot p\cdot \frac{r(r-1)\cdots(r-p+2)}{(p-1)!} \leqq 9^{r+1}\cdot p\cdot r^{p-1}.$$

Since each term in g_{r+1} is $\leqq 1/10^r$, we have, when $r \geqq 2(p-1)$,

$$g_{r+1} \leqq 9p\cdot\left(\frac{9}{10}\right)^r\cdot r^{p-1}.$$